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COGNITIVE ISOMORPHISMS BETWEEN PITCH AND RHYTHM IN WORLD MUSICS: WEST AFRICA, THE BALKANS AND WESTERN TONALITY

This paper compares some diverse musical phenomena in the light of their underlying structural similarities. Specifically, a number of common cyclic structures in pitch and rhythm are found to be isomorphic, and to be understandable in terms of the principles of mathematical group theory. Because such pitch and rhythm patterns are the products of human musical thinking, I call the relationships between them *cognitive* isomorphisms. By this phrase I do not mean to suggest that detailed cognitive models of such patterns are being presented—rather, that the observed structural similarities are sufficiently compelling, and their relation to musical perception and training sufficiently direct, to justify the hypothesis that they may result from general cognitive processes. The basic idea of group-theoretic interpretation is not original here (e.g., see Cassirer, Balzano), and in view of the applicability of group theory to such diverse areas as cognitive and Gestalt psychology, quantum physics, crystallography, change-ringing, general systems theory, Scottish country dancing, decorative ceramic design, knot theory and film animation (e.g., Weyl, Shubnikov and Koptsik, Balzano), cursory mention will be made of related formulations outside the field of music.

To begin with, the perceptual space of interest will be defined in an abstract and general form. Consider a one-dimensional array of lattice sites potentially unbounded in extent, with an 'equivalence operator' operating on the array so that sites L units distant from one another are in some way equivalent. (For example, if these lattice sites are the chromatic scale tones of Western music, then L is 12 and the equivalence operator is the octave.) If M objects (M < L) are then distributed among any L adjacent lattice sites, we obtain a 'unit cell' pattern which may be replicated over the entire lattice. (For example, the white keys of the piano form a unit cell pattern.) These unit cells may operate as a kind of underlying reference pattern for other occupancy patterns overlaid on the lattice. Because of the equivalence operator, the system is no longer most aptly represented as strictly one-dimensional. In fact, Shephard has shown, using pitch as his domain of concern, that a double-helix structure based on two independent variables of pitch height and tone chroma provides a very good geometrical representation of many properties of such a constrained lattice.

Now, to the extent that this kind of scheme corresponds to a general cognitive phenomenon, we should expect to encounter equivalent unit cell structures, for given M and L, in different but equivalently constrained perceptual domains (e.g., musical rhythm, pitch, one-dimensional visual patterns, poetic meter). We might also expect to find predilections for specific values of M or L, or for some particular relation between them, and to be able to formulate general principles of sequential organization of naturally occurring unit cells.

To investigate this in more detail, consider the phenomenon of music, with L = 12. Here it is possible to compare the rhythmic organization of much West African drum ensemble music (that in 12/8 meter) with the pitch organization in Western art music (12 semitones per octave). In the above terminology, the respective lattices consist of discrete points along the axes of time and frequency (notes), the respective equivalence operators are the regularly recurring rhythmic cycle and the octave, and the respective unit cells are the repeating bell patterns (see below) and the Western scale choices. The unit cells in the two musics may be considered to function analogously or be equivalently constrained, in that the bell pattern serves as a time-line reference grid for overlaid rhythmic patterns, and the Western scale serves as a pitch reference grid for the melodic choices of tonality or modality.

Some background information on West African drum ensembles is probably appropriate at this point. Much of this music consists of overlays of various rhythmic strata. For example, in music of the Ewe people of Ghana, a stratum of high-pitched instruments, typically the drum *kagan*, the rattle *axatse*, and the double bell *gankogui*, along with clapping, play virtually unvarying patterns which form a stable basis over which the other drums *kidi*, *sogo* and *atsimevu* play more variable patterns. *Kidi* and *sogo* may be considered to form a second stratum, and normally follow changes in *atsimevu*, the master drum, which is itself a third stratum. Singing and dancing go on simultaneously. The prime responsibility for time-keeping is acknowledged to lie with the bell player (Jones 1959; Locke; C. K. Ladzekpo), and all patterns are fundamentally conceived in relation to the bell, although perhaps also in relation to a basic pulse and other interlocking drum parts in the ensemble. The rattle *axatse* often plays the same pattern as the bell, or a variant based on it. *Kagan* and the clapping usually serve to reinforce the division of the bell pattern into numbers of equal parts. Because of the special acknowledged role of the bell, and its timbral differentiation from the other instruments (it is the only metallophone), it seems valid to consider it as an underlying time-line with only occasional references to other parts. Where bell patterns do not so function, as in the *gamamle* of Ewe music (many bells), this isolation principle may fail to hold. The word 'may' is appropriate here, as even when there is no bell, another instrument (or sometimes clapping) typically performs its time-line organizing function with a similar or identical rhythmic pattern (for example, in Yoruba sacred music of Ekiti, this is the talking drum *kànàngó*—cf. King).

Among the Ewe, whose music is probably the best understood of this region, one bell pattern predominates, namely |♩ ♩ ♪♩ ♩ ♩ ♪| . Its 12/8 structure may be represented in terms of note durations (strictly speaking, time between bell strokes, as in some other patterns the bell is damped) as 2212221. Furthermore, this same cyclic structure is preserved with all possible starting points with respect to the pitch domain in modal music of the medieval European period, natural minor tonality of the classical era, impressionism and centric 20th century music from jazz to the symphonies of Roy Harris, and with respect to rhythm in the time-line patterns of other tradional musics of West African origin.[1] So all the properties of the major scale, such as the cycle of fifths, or isomorphic tetrachord construction, may potentially be found mirrored in these African time-lines.

A comparison of modal treatments of pitch and rhythm for this M = 7, L = 12 case is given in detail in Table 1.* By 'mode' a unit cell structure with specified starting point is meant. Examples of Western music with the indicated pitch structures have not been given as it is assumed the reader is familiar with this repertoire. Included are *all* the time-lines I have been able to glean from a search of the literature and from personal study with several West African drummers which feature 7 strokes in a cycle of 12 beats. Modes of the major scale structure overwhelmingly predominate, and examples of all 'rhythmic modes' except phrygian have been found. Pattern 8 is different, being a mode of the ascending melodic minor scale, but like the other patterns is made up of only the elements 1 and 2. Pattern 9 is from a juvenile song and does not correspond to a common Western scale.

Why is the 2212221 structure so special in both pitch and rhythm domains? In pitch representation, this is an issue with a long and hoary tradition of explanations and diatribes; before delving into this, it will be useful to look at other representative 12/8 bell patterns in the music of this region. In Table II a number of these are displayed. (Two of the patterns—numbers 5 and 6—are from southern Africa, but they seem closely related.) Among these, pattern 3 (22323) is considered particularly important (Jones 1959, 1965; Ekwueme) due to its high incidence and its ability to readily generate, by simple rules of transformation, almost all the other 12/8 patterns in the table. For example, the 4th (23223), 5th (32322) and 6th (32232) patterns follow from it by a shift of modal starting point, and pattern 4 is sometimes considered equally fundamental with pattern 3, due to high incidence. Other patterns may be seen to be related to pattern 3 by the transformation rules of element fusion or fission that correspond to the common musical processes of elision or filling-in. Thus pattern 1 may be derived from 22323 by fusion of the two 2's and cyclic permutation, pattern 9 by fission of the second 3-element into 1 + 2, pattern 7 by fusion of the two 2's and fission of one 3, and the basic pattern 1 of Table I (2212221) by splitting both 3-elements, respectively into 1 + 2 and 2 + 1. Pattern 10 is also so understandable, as a composite of the standard 7-stroke pattern and a steady 4-pulse. Strokes 1, 3, 6 and 8 of this pattern are made on the lower bell of the *gankogui*, and no others are, which supports this interpretation.[2] Patterns 8 and 2, which are not particularly common, may only be derived from 22323 by allowing element permutation as a transformation operation. This is a more

* Tables I–V are to be found at the end of this article.

drastic type of transformation, perhaps particularly plausible where rhythmic organization is additive.

The pattern 22323 is, in the pitch domain, none other than the anhemitonic pentatonic scale CDEGA which has such far-reaching distribution in world music. In West Africa, forms of this pentatonic also occur widely (Nketia 1963a), along with a number of others, notably CEFGB and CEF#GB (C. K. Ladzekpo). Intonation naturally differs from Western norms. West African pitch organization when not pentatonic is most often heptatonic, with non-tempered versions of modes of the major scale used (Nketia 1963a).[3] In view of the transformation rules, the isomorphisms generated from this one basic pattern seem far-reaching, and correspondences for the others are alluring: II – 1 is the minor seventh chord, II – 2 is the dominant seventh chord, and II – 10 is a kind of blues scale. Only the two Ashanti patterns II – 7 and II – 9 do not have common pitch correlates.[4]

Further examples of forms of 22323 in the rhythm domain may be found in other musics with consistent unequal subdivisions of the 12-cycle. Macedonian and Bulgarian folk songs in 12 are known to use the following rhythmic subdivisions: 32232, 32322, 22323, 23223, 32223 (Singer, Kremenliev). Here 32232 is perhaps most common, occurring primarily in western Macedonia. Once again the transformations of applying different starting points and (in the last case) element permutation can simply account for different forms of the same basic pattern. Interestingly, of the five cyclically permuted forms of 22323 possible, all occur here except 23232, which is the only mode of the five to be absent from the African music of Table II as well. It is also a fairly rare mode of this formation in pitch space. This may be due to the fact that only this pattern neither starts nor ends with a 3. This will be discussed further in a more general context below.

Another one-dimensional space which bears some relation to the model used here is that defined by poetic meter. For example, the structure of traditional Japanese verse (Haiku, Waka, Reng) is dominated by constructions of alternating 5's and 7's, which allows partitioning which bears resemblance to patterns already found here. And in fact the work of Halle and Keyser on the analysis of English stress patterns shares a number of general features with the present model. Their analysis consisted of the identification of abstract metrical patterns (usually made up of 1 or 2 kinds of elements), followed by the discovery of rules of correspondence which allow a particular example to be seen as an instance of an abstract pattern. However, I do not wish to go more deeply into this now, partly because poetic meter is often not metric in the same recurring way that music is (with words, the perceptual lattice can be stretched like a rubber membrane) and hence the details of rules of transformation differ.

A transformation rule not mentioned so far, but of great generality, is the idea of complementation. That is, if a pattern is perceptually fundamental, so must be its complement, which psychologically merely corresponds to a figure-ground reversal. Complement here is used in the sense of set theory, that is, the complement of a set of all (unit cell) elements not in S. The importance of the complement is a time-honoured principle of Gestalt psychology (Koffka), and although it is primarily associated with vision in that literature, it has recently been used as a basis for understanding the perception of rhythmic patterns as well (Preusser *et al.*, 1970; Garner 1974; Oshinsky and Handel 1978).

It is quite clear that complementation and figure-ground relations are critical in the performance and perception of West African drum ensemble music. A simple example is the very common Ewe *kagan* patterns 5 – 8 of Table V, which is conceived in complementary relation to a basic 4 pulsation. And often, as for example on the Ewe drums *kidi* and *sogo*, an accented pattern of bounce strokes is filled in completely with muted strokes made by pressing the sticks to the drum head, to help maintain time precision: the mute strokes are then the complement of the bounce stroke pattern, and though not heard prominently in performance, they constitute a strong kinesthetic presence. Likewise, in other hand percussion techniques where the pattern accents are played completely with one hand, the other will fill in the pulses to create a complementary motor image (e.g., some versions of Latin *conga* technique).

In the context of pitch, the pentatonic and major scale structures are complements to each other, as the white and black key groupings of the piano keyboard are known to demonstrate. Hence |♩ . ♪♩ ♩ ♩ ♪| is not only derivable from |♩ ♩ ♩. ♩ ♩. | by fission of the two 3-elements, but perhaps more fundamentally by complementation or figure-ground reversal as well. This sort of derivation emphasizes the holistic nature of the patterns, rather than the values of or interrelations between their components. Picking unambiguously between these two strategies of explanation is hampered by their equal success, though complementation is perhaps more conceptually economical.

An independent musical tradition sheds some light on this issue. In 20th century music by Schönberg, Webern, Babbitt and others, sets of pitches and their complements play a prominent role. In the theory supporting this music (cf. e.g., Forte), it is shown that any set of notes and its complement have important structural similarities which facilitate their integrated usage.[5] This commonality of lattice structure-based complements has important ramifications outside of music as well, as for example in the highly important Ising model (Ising, T. Hill) of statistical physics, where the symmetry of the two-particle distribution function in ferromagnetic or lattice gas systems at complementary densities, based on particle-hole symmetry, is proven for any number of dimensions and may be shown to be equivalent to and historically precede all the other examples given here so far.

Additional insight into the M = 5 and M = 7 patterns given here may be had by a comparison with African bell patterns based on L = 16, another common cycle length in West Africa. (Otherwise, only 8- and occasionally 24-cycles are at all common.) In Table III a number of typical patterns are displayed; despite the longer cycle length, M = 5 and 7 predominate here as well, and all patterns except the last may be seen to contain exactly two 3's. Probably the most widespread pattern is number 1 (33424), which is the basic clavé beat considered to underlie virtually all the 4/4 Afro-Latin music of the Americas. Its basic collection of subunits may be used to derive patterns 2 and 3 by element permutation, and all the other patterns by permutation and fission of elements. The clavé pattern may also be compared to the basic 5-

stroke pattern of 12/8 by aligning the two patterns synchronously and expressing durations in units of 1/48th of a cycle, the lowest common denominator. This yields

8 + 8 + 12 + 8 + 12/48
9 + 9 + 12 + 6 + 12/48,

and it is evident that the two patterns partition the space in very similar fashion, despite their differing cycle lengths. In fact, beginning Western players often confuse the two, for their maximum time discrepancy (occurring at the 4th stroke) is of the order of 1/20 of a second at fast tempos. It may be readily shown by comparison with hypothetical alternatives (e.g., 33334, 32434, 33244) that the clavé pattern is one of the two closest approximations to 22323 that are possible in an L = 16 cycle (4/4 meter). The other pattern, 32434, is a permutation of the same elements. This kind of relation, which I am going to call an example of *analogue* transformation, is a rule of correspondence between patterns of unequal L values, and may be represented approximately by coding the patterns in terms of elements of long (L) and short (S) duration, as has been done in Table III. This form of coding, though too imprecise for many purposes, may be useful for others. Singer, for example, in studying Macedonian dance steps, has profitably used such a representation, and it might be useful for scale characterization.

Analogue transformation between patterns may seem abstract, but it is not merely a theoretical supposition. African and Afro-Latin percussionists generally recognize that some patterns from 12/8 and 4/4 meters may be used in either rhythmic framework (e.g., C. K. Ladzekpo, Pertout). Furthermore, a number of the most clearly African-derived Latin percussion patterns exist in alternate 6/8 or 2/4 versions, or have such versions of the same pattern sounding simultaneously. This is found, for example, in the Cuban *guaguanco* and *bata* drumming styles, in Brazilian *candomblé*, and in the *rhumba columbia* (Pertout).

Next I would like to look at examples for a very different L value, namely L = 7. This value is important because examples of pitch and rhythm structures may be readily compared.[6] Here potential sources include, rhythmically, music of Macedonia, Bulgaria, Turkey, Persia and India, and in the pitch domain, Thai court music, music of the Chopi of south-east Africa, 'Are'are music of the Solomon Islands (Zemp), and diatonic structures of Western music. Thai court music is known to be based to reasonable approximation on a tempered 7-note system (Morton 1976), and this is also the tuning ideal for the Chop xylophone music, as revealed verbally by the musicians and supported by averages of xylophone tunings (Tracey). The inclusion of diatonic Western structures is based on the perceptual rather than frequency equivalence of the 7 tones in the scale. That this is so for the lay listener was pointed out to me by Roger Shepard, and is supported by research in Gestalt psychology (Koffka) and categorical perception (e.g. Siegel and Siegel).

Table IV lists the common 7-beat patterns of Balkan music and compares them to Western diatonic structures. The patterns given comprise all structures of fewer than 5 strokes which are composed of nearly equal elements of only 2 sizes (primarily 2 and 3 or 1 and 2; patterns of 1 and 3 are not common). A similar situation obtains with Indian *talas*. The isomorphisms between the two spaces are again rather far-reaching, since the given isorhythms define all the basic harmonic components of traditional Western tonal music: tonic-dominant relation and inversions of triads and seventh chords. The same principles of structural transformation found for the L = 12 case seem to hold here, as well: cyclic permutation of pattern 2 (223) serves to generate patterns 3 and 4, and fission of the 3-element into 2+1 or 1+2 can account for 5, 6, 7 and 8. Once again, the principle of complementation may equivalently be considered to be at work, for the M = 3 and M = 4 patterns are complements of each other, as the reader may verify.

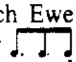
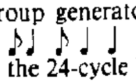
Although Chopi and Solomon Islands music seem too poorly understood at this time to form the basis for this sort of analysis, Thai music has been documented in some detail by Morton (1968, 1976). However, comparison with Thai music requires caution, as some natural criterion for identifying the unit cell pitch patterns needs to be invoked: there are no 'chords', for example, to unambiguously link sets of pitches. Typically it is found that melodies use 5 of the 7 tones, or, in the *mǫn* style, 6 tones or even all 7. However, these tones are not used with equal frequency or emphasis, nor are all used within a given phrase. The importance of the Thai 5th relation is quite general (corresponding to pattern IV – 1), and the tones falling on the open and closed strokes of the *ching* (small finger cymbals) are considered to receive special emphasis. A typical example is the piece *Nok Khao Khamae* (Morton 1968), where only tones 1, 3, 5 and 6 occur on *ching* strokes. These may be seen to form pattern IV – 6 (2212), and a number of other pieces also yield basic patterns of Table IV. On the other hand, in the piece *Khamēn Saiyōk*, synchronous strokes of *ching* and gong emphasize first tones 1 and 6 (as in IV – 1), and then tones 2, 3 and 6 (Morton 1968). This second case may be understood as a stacking of Thai 5ths, generating a 313 structure which we did not encounter in the Balkans, though it has a pitch correlate in the common suspended 4th chord of contemporary jazz. So here the isomorphism is useful, but not as far-reaching; it is possible that my limited understanding of Thai music is at fault. (Metrically, Thai music is always 2/4 or 4/4, with the exception of a body of songs in 7/4, with *ching* strokes divided 2212 [Moore].[7]

Interpretations: Group Theory and Perception

It seems appropriate to consider why these few special patterns[8] have been chosen by so many apparently independent musical cultures. It is not an easy question. The answer attempted here is based on simple mathematical modelling and the stance of the cognotive psychologist; I have the belief that this commonality must tell us something underlying about perception and the mind.

Perhaps the most elegant derivation of the widespread 22323 and 2212221 patterns in the pitch domain has recently been given by Balzano, whose ideas were seminal for the interpretation offered here. While acknowledging other special historical sources for our tonal structures, such as vibrating body over-tones, the tetrachord and the desire for modulation, he finds the special properties of the pentatonic and major scales in our 12-note system derivable from mathematical group theory. In particular, our musical system is an example of the abstract group C_{12}, and it is expressible as the direct product of two smaller subgroups as $C_3 \times C_4$ (e.g., Budden). A consequence of this reducibility to subgroups is the natural representation of the major scale structure as a convex region in a two-dimensional lattice with unit cell dimensions 3 $\times$ 4; 3 and 4 semitones are the pitch distances used to define triads, of course. An important and natural feature of this group theoretic representation is the differentiation (for L = 12) of patterns based on the distances 2, 3, 4 and 6 from patterns of other lengths, since these numbers, being factors of 12, will not generate all elements of the group if replicated, but rather reproduce the same (6, 4, 3 or 2 respectively) elements after the distance of one cycle. In contrast, the numbers 1, 5, 7, 11 are group generators of C_{12}, because they are prime with respect to the group order (12), and will generate all elements of the group before repeating any (Boretz 1971). In musical space, the numbers 1 or 11 just generate the chromatic scale or underlying rhythmic pulse, while 5 and 7 generate the cycle of 5ths: four 5ths generate 22323, while six 5ths generate (the Lydian mode of) 2212221. In rhythmic terminology, polyrhythms are built up from subgroups, and time-lines are formed by replicating a non-unit generator.

If this same procedure is tried for L = 7, similar results are achieved, though here the group order (7) is a prime number so that all numbers less than 7 are generators. Since there are no subgroups there are no polyrhythms, and the replication of 2, the smallest non-unit generator, generates 223 and 2221, the fundamental structures for this size cycle. Structures generated by 3 are not common in the musics under discussion.

The case of L = 8 may also be mentioned at this point for completeness. Although pitch space data seem to be unavailable, rhythmically such Ewe pieces as *Kinka* and *Sovu*, and the Lobi popular music *Nandon*, utilize  or 332; the samba, habanera, tango and innumerable other African-influenced Latin American dance forms use it as well. (Although two of these cycles may be considered to form a cycle of 16, I have not done this here, preferring to take the smallest *recurrent* unit as fundamental.) In any case, the smallest non-unit group generator is 3, and generates precisely this pattern. Its complement,  , may also be generated by 3, and is the basic replicating unit in the 24-cycle bell pattern of the Dahomey piece *Kadodo* (Combs 1975). This complement is also widespread in Afro-American music (Evans), and is used with displaced starting point as time-line for the Ewe piece *Adzro* (Fiagbedzi) and the Ashanti piece *Kumachacha* (Combs 1974).

For L = 16, where again we have only rhythmic information, the smallest non-unit generator is again 3, which yields (on 4 replications) the pattern 33334 or  , which is almost but not quite the fundamental clavé pattern.[9] The difference is slight, for 4 of the 5 strokes are identically positioned, and the other, the 4th, differs by only one semi-quaver.[10] Nevertheless this discrepancy poses a problem for the group theoretic derivation of pattern commonality offered here. The only plausible explanation seems to be based on ascribing this difference to the perceptual pull of the predominant 12-

pattern 22323, using the idea of analogue transformation presented earlier. That is, the 'intrinsic' pattern 33334 has been stretched slightly to become the 33424 clavé rhythm on the basis of the latter's close perceptual analog relationship to the widespread 22323 pattern.

This explanation in terms of underlying cognitive structures and simple group properties might be expected to apply to aspects of the music other than bell patterns and diatonic scales. For L = 12, where this possibility can be most clearly tested, it does seem to hold.

The basic African 2-, 3-, 4- and 6-clap cycles are respectively equivalent to the tritone, the augmented triad, the diminished 7th chord, and the whole tone scale. In both pitch and rhythm space, the 4-cycle is in some sense the most fundamental of these possibilities: the 4-clap constitutes the 'basic pulsation' (C. K. Ladzekpo; Locke), while the diminished 7th chord is a mainstay of modulation for tonal music. Extensive use of 3- and 6-clap patterns as well in African music points out that its best pitch analogs are late romantic, impressionist, early 20th century music, and of course, contemporary jazz. In fact, just the scales used in jazz (Pressing) are particularly well represented in West African rhythm space, as may be seen in Table V. Since pitch and rhythm domains are subjectively so different (serialist music notwithstanding), it is perhaps surprising that these structural analogies should go as far as they do.

The differences that do exist provide some interesting information. For instance, in rhythm space the major triad structure does not turn up prominently for L = 12 as it did for L = 7. This seems to support the idea of the triad as a diatonic scale-derived concept based on synchrony; it may also be due to the low filling ratio value this corresponds to (see below). And two scales occurring in jazz which do not, to the best of my knowledge, correspond to common West African patterns, throw additional light on the matter. Harmonic minor (2122131) and harmonic major (2212131) scales are the least common jazz scales. They are the only scales here which have elements of three sizes (1, 2, 3); hence they are in some sense less economical in their use of material, and sample the space more unevently. Also, each of these contains the diminished 7th chord, which corresponds to every on-beat of some underlying basic 4-pulsation. Since the time-lines are *already* perceived in composite relation with the underlying 4-pulse (Locke; C. K. Ladzekpo), these patterns may lack the perceptual flexibility or variety required of a maximally versatile time-line.[11] In fact, one of the interesting properties of the 22323 and 2212221 structures, *not shared by any other structures here*, which supports such an idea, is their property of sampling as equally as possible the various subdivisions (subgroups) of 12: 2, 3, 4 and 6. Mathematically this occurs because the pattern (group) generator 5 (or 7) is prime (shares no common factors) with respect to these numbers. For instance, there are only 2 distinct 6-cycles, which interlock, as indicated by the following pattern of x's and o's:

cycle 1
xoxoxoxoxoxo
cycle 2

(In pitch space these are the two complementary whole tone scales.) The 22323 and 2212221 structures, regardless of at what point they are begun, when overlaid on this, divide their strokes as nearly equally as possible between the x-and o-cycles; e.g., 3(x) + 2(o) and 4(x) + 3(o), respectively. That is, three strokes of the 22323 pattern will fall on elements of one 6-cycle and two on elements of the other 6-cycle; four strokes of the 2212221 pattern will fall on elements of one 6-cycle and three on elements of the other. A similar equal partitioning occurs for the three 4-cycles: xo – xo – xo – xo – ; 2 + 2 + 1 and 2 + 2 + 3 respectively; and the four 3-cycles: x + o – x + o – x + o – ; 2 + 1 + 1 + 1 and 2 + 2 + 2 + 1. And all six 2-cycles produce the respective sampling patterns 1 + 1 + 1 + 1 + 1 + 0 and 1 + 1 + 1 + 1 + 1 + 2. The musical usefulness of this equal sampling distribution generated by the fundamental patterns may be very great. Suppose the African master drummer wishes to play a pattern based on one of these cycles, with specified starting point. This sampling property means that there will always be several bell strokes he can tune into which will not only locate the proper starting point, but generate the correct rhythmic feel as well. I do not know whether such a process goes on consciously, but it must facilitate the aggregation of underlying pulsations into perceptual units of various sizes, which is so typical of master drum patterns (see Locke; Jones 1959).

This idea of equal sampling may be used in another way as well—in understanding the choice of M for a given L. Let s = M/L define a rhythmic filling ratio, in the manner that von Hoerner has done for pitch. Then, as in his work, one would predict that this number should not be too far from ½, so as to sample the space adequately, and make maximal use of figure-ground relations.[12] Such a prediction also follows from the theory of bisection proposed by Rahn (1977, 1978a). In practical terms, this means that elements of size 2 will often predominate. And the most common patterns looked at here yield: for L = 12, s = 5/12, 7/12; for L = 7, s = 3/7,4/7; for L = 16, s = 5/16, 7/16, 9/16;[13] and for L = 8, s = 3/8, 5/8. These numbers support the idea quite strongly, except for s = 5/16, which seems to confirm the specialness of the corresponding clavé pattern and support its derivation from 22323 by analogue transformation. Two other bell patterns which support s ≈ ½ are the L = 24 *Ago* (s = 11/24) and *Kododo* (s = 15/24), part of the suite *Adzohu*, from Dahomey (C. K. Ladzekpo; Combs 1975).

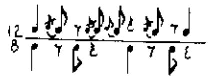
A noticeable feature of these s ≈ ½ filling ratios is that, particularly when L is even (12, 16, 8), M is almost always odd; notably, the exact middle value, the even number L/2 seems to be avoided, except in some Ashanti music (pattern 2 – 9, and even there the 6th stroke is often omitted). Why this should be so is not immediately apparent from the group theory explication given here; it may have a base in aesthetic preferences or special meanings given these numbers (especially 5 and 7) in these cultures (cf. Zaslavsky for special properties of African number systems).

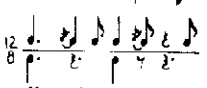
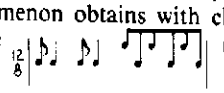
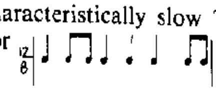
At this point a look at the constraints, both cognitive and kinesthetic, imposed by the physical act of performing these rhythms seems appropriate. For, unless all the constraints imposed on the basic perceptual space correspond in different representations, it is unlikely that similar structural choices will be made.

It seems quite significant that the tempos of West African and Bulgarian or Macedonian music are all relatively fast. *Nyayito*, one of the very slowest Ewe pieces, moves at something like ♪ = 226, and tempos for fast pieces like *Atsiabeko* are ♪ = 600+, close to the possible performance limit. Likewise, Bulgarian and Macedonian tempi are quick, and if ♬ is taken to be the basic fastest pulse, range most commonly from ♬ = 250–500. Considering these speeds with respect to the finger, wrist and arm movements required in performance suggests that the quickest pulse functions both as a cognitive grid and a kinesthetic threshold, which may be decorated occasionally in the service of heightened rhythmic activity, but whose underlying basis cannot be easily controverted.[14] This kinesthetic threshold facilitates the cyclic time-line idea in the same way that instrumental design and physical playing habits reinforce any but the most determined Western microtonalists from making subdivisions of 12-tone temperament more than occasional features of their compositions. And likewise, fast tempos would be expected to limit structures composed of strings of 1's. In fact sequences of 1's longer than 2 are not found in Macedonian dance (Singer), and even adjacent 1's are absent from all the basic African time-lines. (The Ewe *Nyayito*, which is a rare exception, is comparatively very slow and may be considered a composite pattern, as noted earlier.) This same property is found mirrored in the pitch domain for different (harmonic) reasons: Pressing has shown that the avoidance of adjacent half-steps is a principle which underlies the construction of jazz scales.

Fast tempos also facilitate the focusing of attention on the cycle as a whole, rather than only on serial relation; again, this fact argues for patterns with holistic group transformational properties. World musics with characteristically slow tempi and cycles of 12 do not, in fact, use the basic time-lines found here.

Consider, for example, the basic *kutkŏri* pattern of Korean music,



or *t'aryong*,  played on both ends of the *changgo* drum (Song). The top lines here are respectively 221322 and 321231; the slow tempi do not favour polyrhythms, and these patterns are unlike those found earlier. A similar phenomenon obtains with characteristically slow Turkish music, with patterns like  or  (Darbaz). And African melodic phrase lengths, considered in this cyclic fashion, are not dominated by the divisions of 12 given here, though they do occur (see Ewe children's songs, Jones 1959).

This suggested dependence of pattern on tempo is supported by considerable research in cognitive psychology which finds different kinds of perceptual organization operating on different time scales. This has been conceived as a linking up of internal time scales with the time scales of external events, as described in the work of Mari Jones. Other examples are the demonstrated importance of tempo on the perception of single line rhythms (e.g., Preusser 1972), and also on the perception of polyrhythms, as reported by Oshinsky and Handel (1978) and Handel and Oshinsky (1981). They found, for example, that 2 × 3 polyrhythms were perceived predominantly as triple meter at slow tempos, but predominantly as duple meter or a composite pattern at fast tempos. How such findings might differ for African listeners, with their tradition of composite 2 versus 3 hearing, is an interesting question.

Mention should be made at this point of the work of Rahn (1977, 1978a, 1978b), which can provide the basis for an alternative explanation of the structural similarities presented here. His theory of bisection allows the generation of nearly all the patterns given, and must be carefully considered as an alternative. The only disadvantage which may be pointed to is the fact that the number of possible patterns it generates is far greater, and hence its predictive efficiency in explaining these particular widely occurring isomorphic patterns is less than that of the current group theoretic derivation.

Perception of Temporal Structure

Also germane to the present theory are the results of recent research in cognitive psychology on the perceptual organization of temporal patterns. Here important contributions have been made by Fraisse, Royer and Garner, Handel and Buffardi, Preusser *et al.* (1970), Handel (1973, 1974), Restle (1970, 1972, 1976), Garner (1974), and Handel and Todd (1981), among others.

Experimentally typical and of particular relevance to the theory presented here is the work of Garner and co-workers. Their experiments entailed presenting repeating sequences of dichotomous elements to subjects and asking them to reproduce the patterns. Most of the work was done with 2-element patterns of varying lengths, with the elements represented by high and low tones, or red and green lights. The information obtained included the preferred perceived starting point(s) of the recurrent structures, and relative difficulty of learning the patterns as measured by the number of cycles required to learn them. In this form the data might be applied most directly to the two-toned bounce and mute strokes of the Ewe drums *kidi* and *sogo*, but this will not be done here.[15] More significant for the time-line structures of this paper is the result that the preferred perceptual organizations of the 2-element patterns proved to be understandable in terms of organization of the constituent complementary 1-element patterns, a representation equivalent to that of the time-line (Preusser *et al.* 1970; Garner 1974). Basically it was found that the difficulty of pattern perception corresponded closely with the number of preferred perceived starting points in the endless cycle presented, and that runs of successive elements were seldom, if ever, broken up (Royer and Garner 1966). Furthermore, possible perceived starting points (hence perceived difficulty) could be accurately predicted by the interaction of two principles:

1. Pattern descriptions tend to end with the longest gap between elements (gap principle);
2. Pattern descriptions tend to begin with the longest run of elements (run principle).

When the two principles agree, learning the pattern is easy and unambiguous; when they disagree, pattern learning is more difficult, and a variety of starting points is perceived.

For example, consider the clavé rhythm, 33424: there are no runs of adjacent elements (♫), so that principle 2 is not operative, whereas the gap principle predicts that the pattern should end with one of the two 4's, as it does. The application of these principles to time-lines requires some caution, however, for, first, a composed pattern can be *learned* with any starting point; second, it is possible that these psychological data are culture-bound; and third, the time-line is part of an ensemble and not the only important aural stimulus. In addition, as mentioned above, many time-lines given here are conceived in relation to an underlying pulse, each beat of which spans 3 or 4 smallest time units; because of this additional perceptual constraint, these rhythmically divisive instances are less completely isomorphic to the psychological experiments under discussion than are cases of purely additive organization (e.g., Balkan rhythm, Western scales). Consequently, while many important time-line structures here *do* follow the rules postulated above (e.g., 22323, 23223, 33424), many others, notably those with preparatory upbeats (e.g., 2212221), do not, and the issue may be further complicated by the philosophical idea (as among the Ewe) that the start and end of a pattern can be the same.

Examination of the time-lines given here seems to indicate that an extension of Garner's two principles to account for the possibility of perceptual aggregation is required, especially in musical context. Such perceptual aggregation is common knowledge among performing musicians, and has been described recently in some detail by Handel and Todd (1981). Specifically, perceptual subunits may be construed, facilitated by the organization of the pattern and an underlying pulse (if present), which consist of one or more elements and one or more gaps: the additional principle proposed here then states that pattern descriptions tend also to avoid breaking up runs of such perceived aggregate subunits. For example, in this way of thinking the basic pattern 22323 may be considered to be made of unequal 'elements' of 2's and 3's, rather than single elements and gaps. This is facilitated by the complete absence of 1's from the pattern, and would provide the prediction that 4 of the 5 modes should be perceptually well organized; only 23232 would break the single run of 2-elements and hence be a poor time-line choice. As the reader will recall, just this situation obtains for the West African and Balkan musics here. Likewise, the preferred starting point of Ashanti pattern I – 8, 2121222, is readily understood from perceptual aggregation as 2121222, with the run of 2's and the run of aggregated 3's remaining unbroken. Similarly, the invoked third principle may be considered to explain the rarity of 232 in Balkan music, and is consistent with the preferred starting points of all the L = 16 patterns of Table III, which do not follow Garner's first two principles alone.

The only remaining patterns whose starting points require further explanation are 2212221 and 1222, where in each case all modes may in principle occur but those beginning with 1 are rarest (compare phrygian and locrian modes, and Table IV).[6] The following explanation can be offered in light of principle 3: both sequences are dominated by runs of 2's which are broken up, or rather shifted, by the 1's. To have the start of a cycle begin with 1, the short shifting element, without a possibility of perceptual aggregation, is jarring enough (at quick tempos) to make those patterns rare as reference structures. (An explanation in pitch space would likely need to invoke principles of tonality or leading tone.)

A related perspective on perceived pattern difficulty may be seen in the work of Royer and Garner (1966), who carried out a detailed study of all possible 2-element patterns of length 8; unfortunately, no comparable examination of patterns of length 12 has been reported. For L = 8, although $2^8 = 256$ patterns are in principle possible, only 16 2-element patterns are actually independent and not reducible to smaller cycle lengths. Of these, two were by far the most perceptually ambiguous and difficult to learn for subjects: xxoxxoxo and xxoxooxo. In terms of the constitutive 1-element patterns (either x's or o's may be chosen as the foregound, with the other symbol supplying the gaps), these are respectively 12122 or 332, and 1232, and are exactly the 3 patterns produced by the C_8 group generator, 3. That is: $3^2 \rightarrow 332$, $3^3 \rightarrow 1232$, $3^4 \rightarrow 12122$. Note that $s = M/L \approx \frac{1}{2}$. Because these patterns may be so generated, they sample the 2- and 4-cycles of this group uniformly, and their perceptual multistability may be considered to reflect their unique derivation from the group generator. The first two patterns, as 332 and its complement 12122, have been shown to dominate West African (and Afro-Latin) 8-cycles earlier, while the third is rare, again an example of the avoidance of $M = L/2$. Despite the clear need for analogous investigation of the case L = 12, it seems that these data from cognitive psychology strongly support the idea that multistability of perceptual organization is of crucial importance in the design of West African rhythmic patterns. Such an aesthetic has been alluded to by previous authors (e.g., Locke). If true, it is an aesthetic exactly opposed to the Gestalt idea of simple symmetry and 'pattern goodness' (Garner 1970). Such multistability will allow physically repetitious parts to be perceptually changing and interesting for players of fixed rhythms, and serve to allow variety of starting point and pulse conceptualization for players where variation and improvisation play a greater role (e.g., master drummer).

OVERVIEW

At this point I would like to summarize the main themes of this paper. These are:

1. Musical pitch and time structures, under suitable restraint conditions, may be seen to be isomorphic, and correspond to a partially filled one-dimensional cyclic lattice which geometrically can assume the shape of a spiral or double helix (Shepard). Suitable restraint conditions are, in pitch, octave equivalence and perceived equality of smallest intervals, reinforced by instrument design, and in time, repeating rhythmic cells (isorhythms) based on a uniform fastest unit, reinforced by quick tempos and kinesthetic thresholds of performance.

2. Let cycle size be labelled L, and number of occupied sites in this unit cell be M. Then L = 12, M = 5, 7 are the values which generate identical struc-

tures in West African time-lines and scales, Western tonal music (including jazz), and Balkan rhythm. L = 16 structures of West African and Afro-Latin music may be understood to be transformations of a basic 33424 strucrure, while for L = 7, identical patterns are observable in Bulgarian and Macedonian music, Western diatonic structures, and to a lesser extent, in Thai court music.

3. All the patterns found may be derived from a few basic patterns (primarily 22323 (or 2212221), 322, 323) by one or more of the following rules of transformation:

(a) cyclic permutation
(b) element fission or fusion
(c) complementation, or figure-ground reversal
(d) maximally similar analogue approximation, as for example the relation of 22323 to 33424
(e) element permutation

Those transformations are more common and basic which preserve the pattern in a holistic sense; in particular, (e) involves structural derangement that is more drastic than the others.

4. The predominant pitch and rhythm patterns used may be readily understood in terms of mathematical group theory. In particular, the cycles form the groups C_L, and the fundamental patterns for each value of L may be generated by replication of the smallest non-unit group generator. For C_{12} this generator is 5; for C_7, 2; for C_{16} and C_8, 3. The subgroups of these groups are found to play important perceptual roles in the music, particularly where L has many independent factors, as for L = 12 (or 24).

5. The work of Garner and co-workers on temporal pattern perception has been examined and found adequate for only a partial explication of the preferred perceived starting point and ease of learning of time-line patterns. To extend this, it has been suggested here that in musical pattern perception, the perceptual aggregation of elements and gaps into larger subunits also occurs, and that time-lines are often designed to avoid breaking runs of these composite subunits. The data appear to support such an interpretation.

6. There is some evidence that the few basic patterns for a given L possess maximal perceptual multistability (of starting point) of all possible patterns, consistent only with the constraint that the filling ratio s = M/L not be too different from ½. (Perceptual multistability seems, in fact, to be a feature of West African drum ensemble music.) This multistability appears to correlate closely with the basic patterns' property of sampling all existing subgroups as equally as possible, a property which follows from their formation from group generators, as given in point 4 above. If L is a prime number, so that C_L possesses no subgroups (e.g. L = 7), then the concept of subgroup sampling is not applicable, and may be replaced by sampling runs composed of units of small size (2, 3 or 4).

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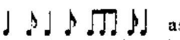
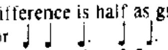
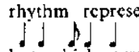
1 These patterns are actually distributed even more widely than West Africa, and a few examples from southern Africa are included to support this notion.

2 This same 9-stroke pattern occurs with displaced starting point  as a superposition of interlocking patterns played on axe-blades among the Bemba people of northern Rhodesia (Jones 1965). Compare also the Ewe piece *Atigo* (Fiagbedzi).

3 That this similarity of structures in West African pitch and rhythm exists points to the idea of preferential apportioning of cognitive spaces regardless of whether an underlying lattice exists, since West African music is not known to be based on a 12-note system. This idea is related to the presence of analogue transformation in rhythmic patterns (see below).

4 Pattern II – 9 may be written alternatively with stroke 3 (♩.) as the starting point. In either case, the stroke given first in Table II is also considered to end the pattern. Also, parentheses around the second stroke indicate that it may be omitted; it corresponds to the so-called 'motor image' (C. K. Ladzekpo).

5 To be specific, this concerns the quantity known as the interval vector for the two sets. This is just a display of all possible intervals between members of a set of notes. It is usually written in terms of interval classes, each of which consists of an interval and its inversion. The theorem states that the difference in respective interval vector entries between a set and its complement is a constant equal to the difference in sizes of the set and its complement, with the exception of the one vector entry which bisects the space, where the difference is half as great. To give an example in rhythm representation, the interval vector for  is [032140] and that for  is [254361]. All entries differ by $7 - 5 = 2$ from each other, except for the last, which corresponds to interval class 6, where the difference is $2/2 = 1$.

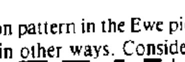
6 Perhaps the only other value where ready comparisons might be made is $L = 5$, with pentatonic scales of Java and East Africa being compared with Turkish and East African rhythms, among other possibilities.

7 Still another example may be found in heptasyllabic couplets of Chinese verse (Jakobsen), which have the structure |– –|◡ ◡ – – ◡|

|◡ ◡|– –|◡◡ –‖, revealing both the idea of complementation and a basic 223 = 2221 structure.

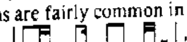
8 A precise minimal set of patterns has not been formulated, since the available musical data are most likely insufficiently comprehensive.

9 This pattern instigates a recurrent relaxation pattern in the Ewe piece *Gahu* (C. K. Ladzekpo).

10 Other patterns belie the 33334 structure in other ways. Consider the Bemba music Imbeni, where the time-line part (played on a drum) is  (Jones 1958), or similar and characteristic syncopation in Afro-American music like ragtime or swing.

11 The Ewe music *Adaϑu*, one of the very rare pieces where a strictly on-beat bell pattern occurs, is very fast and of short duration.

12 Although it would be foolish to base any concrete claims on it, Zaslavsky indicates that the Shambaa language of north-east Tanzania and some Bantu languages utilize number nomenclature based on bisection, for the numbers 6 to 10: $6 = 3 + 3$, $7 = 4 + 3$, $8 = 4 + 4$, $9 = 5 + 4$.

13 Nine-stroke patterns are fairly common in 4/4 time-lines of Afro-Latin music ($s = 9/16$), as in the *guaguanco* pattern , also heard in African highlife bands.

14 In fact a comprehensive treatment of all the rhythmic strata, not just the time-line, would have to include divisions of ♪ into ♬ : that is, a cycle of 24. Since C_{12} is a subgroup of C_{24} (lacking only the 8-cycle), the transition should be straightforward, decomposing C_{24} as $C_3 \times C_8$. This structural possibility is utilized in the two bell patterns mentioned earlier, Ago and Kadodo, which are both generated by the use of 8×3, and in the 8-clap cycle in 12/8.

[15] That is, the support drums in Ewe and much other African music are played in such a way as to produce two distinctly different tones (often called bounce and mute strokes when drums are played with sticks). The resulting patterns could thus be directly compared with the patterns used in this psychological research.

[16] A few Balkan patterns remain elusive, such as the 11/16 cycle divided 22322. These rare exceptions do not significantly weaken the status of the rules, in as much as the rules specify *tendencies* for perceptual organization, not pre-emptive possibilities. Creative individuals have always sought to produce viable exceptions to common musical practice.

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TABLE I

Comparison of M = 7, L = 12 patterns for pitch (scales) and rhythm (time-lines)

pattern	pitch domain name and notation (in C)	rhythm domain notation	examples from West Africa	references
1. 2212221	major scale (Ionian) CDEFGAB	♩ ♩ ♪♩ ♩ ♩ ♪	Ewe (Atsiabek , Sogba, Atsia) also Yoruba	Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke
2. 2122212	Dorian CDE♭FGAB♭	♩ ♪♩ ♩ ♩ ♪♩	Bemba—Northern Rhodesia	Jones (1965), (Ekwueme)
3. 1222122	Phrygian CD♭E♭FGA♭B♭	♪♩ ♩ ♩ ♪♩ ♩	——	——
4. 2221221	Lydian CDEF#GAB	♩ ♩ ♩ ♪♩ ♩ ♪	Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo)	C. K. Ladzekpo, Combs (1974), R. Hill, Asiama
5. 2212212	Mixolydian CDEFGAB♭	♩ ♩ ♪♩ ♩ ♪♩	Yoruba sacred music from Ekiti	King
6. 2122122	Aeolian CDE♭FGA♭B♭	♩ ♪♩ ♩ ♪♩ ♩	Ashanti (Ab fo , Mpre)	Koetting
7. 1221222	Locrian CD♭E♭FG♭A♭B♭	♪♩ ♩ ♪♩ ♩ ♩	Ghana*	Nketia (1963a)
8. 2121222	(#2 Locrian) CDE♭FG♭A♭B♭	♩ ♪♩ ♪♩ ♩ ♩†	Ashanti (Asedua)	C. K. Ladzekpo
9. 2112123	—— CDD#EF#GA	♩ ♫♩ ♪♩ ♩.	Akan (juvenile song)	Nketia (1963b)

* clap pattern
† mute stroke on bell

TABLE II

Other 12/8 bell patterns and their pitch equivalents (M = 4, 5, 6, 9; L = 12)

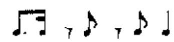
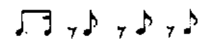
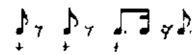
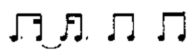
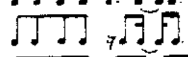
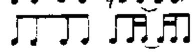
	M	pattern	pitch representation	rhythm domain notation	examples from West Africa	references
1.	4	3234	CE♭ FA♭	♩. ♩ ♩. ♩ 𝄽	Common Ashanti pattern	Koetting, Combs (1974)
2.	4	3324	CE♭ G♭ A♭	♩. ♩. ♩ ♩ 𝄽	Akan (juvenile song)	Nketia (1963b)
3.	5	22323	CDEGA	♩ ♩ ♩. ♩ ♩.	Ewe (Af u), many others	C. K. Ladzekpo, Ekwueme, Jones (1959, 1965) Nketia (1963a), Asiama
4.	5	23223	CDFGA	♩ ♩. ♩ ♩ ♩.	Akan (Akatape)	Nketia (1963a)
5.	5	32322	CE♭ FA♭ B♭	♩. ♩ ♩. ♩ ♩	Venda (children's song)*	Blacking
6.	5	32232	CE♭ FGB♭	♩. ♩ ♩ ♩. ♩	Bemba (main pattern accents)	Jones (1965)
7.	5	32124	CE♭ FG♭ A♭	♩. ♪♩ ♩ ♩ 𝄽	Akan (Adowe)	Coombs (1974)
8.	5	22233	CDEF#A	♩ ♩ ♩ ♩. ♩.	Ga (Kple) Ewe (Takada axatse pattern)	Nketia (1963a) S. K. Ladzekpo and Pantaleoni
9.	6	223212	C(D)EGAB♭	♩(♩†) ♩. ♩ ♪♩	Ashanti (many pieces)	C. K. Ladzekpo
10.	9	211111221	CDD#EFF#GAB	♩ ♪ ♫♪ ♪♩ ♩ ♪	Ewe (Nyayito)	Jones (1959)

* clap pattern
† mute stroke on bell

TABLE III

TABLE III

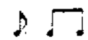
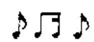
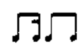
Typical L = 16 West African bell patterns

	M	pattern	rhythm domain notation	short-long representation	examples from West Africa	references
1.	5	33424		SSLSL	Akan, Ga-Adangme, Ewe, Afro-Latin clavé beat	C. K. Ladzekpo, R. Hill
2.	5	33442		SSLLS	Ewe (Gahu), also Nigerian	A. K. and K. Ladzekpo
3.	5	44323		LLSSS	Ga-Adangme (Kpatsa)	C. K. Ladzekpo
4.	7	3232222		LSLSSSS	Ga-Adangme (Takoe)	C. K. Ladzekpo
5.	7	2222323		SSSSLSL	Adangme	C. K. Ladzekpo
6.	7	2223223		SSSLSSL	Ghana*	Nketia (1963a)
7.	9	222221212		LLLLLSLSL	Ga(Oyaa)	Combs (1974, 1975)

* clap pattern
† mute stroke

TABLE IV

L = 7 structures for pitch and rhythm

	M	pattern	diatonic description*	rhythm domain notation	Balkan example	references
1.	2	243	CG: tonic-dominant		Samodiva grád gradila (fairly rare)	Kremenliev
2.	3	223	CEG: root position triad		ruchenitza	Kremenliev
3.	3	322	CFA: 2nd inversion triad		Makedónsko hóró	Kremenliev
4.	3	232	CEA: 1st inversion triad		rare in Balkans, but occurs in Turkish music	Kremenliev, Darbaz
5.	4	2221	CEGB: root position seventh chord		ruchenitza	Kremenliev
6.	4	2212	CEGA: 1st inversion seventh chord		Eleno Mome	Singer
7.	4	2122	CEFA: 2nd inversion seventh chord		Petko Klapejko	D. Golber, (personal communication)
8.	4	1222	CDFA: 3rd inversion seventh chord		none known; occurs in Persian music, however	

* based here on major scale, but any diatonic mode might be used, so that all common types of triads and seventh chords can be generated.

can be generated.

TABLE V

The common jazz scale structures (Pressing) and their rhythmic isomorphisms

	M	pattern	jazz scale	rhythmic equivalent	examples from West Africa	references
1.	5	22323	CDEGA	♩ ♩ ♩. ♩ ♩	basic pattern	see Table II
2.	6	222222	CDEF#G#A#	♩ ♩ ♩ ♩ ♩ ♩	6-clap	Jones (1959)
3.	6	131313	CD♭EFG#A	♫ 𝄽 ♫ 𝄽 ♫ 𝄽	Ewe kidi bounce stroke patterns	C. K. Ladzekpo, Jones (1959)
4.	7	2212221	CDEFGAB	♩ ♩ ♪♩ ♩ ♩ ♪	Agbadza, Af u basic pattern	see Table I
5.	7	2122221	CDE♭FGAB	♩ ♪♩ ♩ ♩ ♩ ♪	axatse in Af u,* bell in Asedua*	C. K. Ladzekpo
6.	7	2122131	CDE♭FGA♭B	♩ ♪♩ ♩ ♪♩. ♪	——	——
7.	7	2212131	CDEFGA♭B	♩ ♩ ♪♩ ♪♩. ♪	——	——
8.	8	12121212	CD♭E♭E♮F#GAB♭	𝄾♫ 𝄾♫ 𝄾♫ 𝄾♫	Kaga in many Ewe pieces*	C. K. Ladzekpo Jones (1959)

* present in modal form; different starting point used here than in scale representation